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Forecasting the Diffusion of Innovative Products Using the Bass Model at the Takeoff Stage: A Review of Literature from Subsistence Markets

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Abstract A considerable amount of research has been directed at subsistence markets in the recent past with the belief that these markets can be tapped profitably by marketers. Consequently, such markets have seen the launch of a number of innovative products. However, marketers of such forecasts need timely and accurate forecasts regarding the diffusion of their products. The Bass model has been widely used in marketing management to forecast diffusion of innovative products. Given the idiosyncrasies of subsistence markets, such forecasting requires an understanding of effective estimation techniques of the Bass model and their use in subsistence markets. This article reviews the literature to achieve this objective and find out gaps in research. A finding is that there is a lack of timely estimates of Bass model parameters for marketers to act on. Consequently, this article sets a research agenda that calls for timely forecasts at the takeoff stage using appropriate estimation techniques for the Bass model in the context of subsistence markets.

Keywords Bass model, takeoff stage, subsistence markets, critical review

I. Introduction

Many innovative products do not succeed in the marketplace despite optimistic projections, one example being improved cookstoves promoted by various governments among the underprivileged. Also, consider the case of alternate-fuelled vehicles (AFV). Despite many encouraging sales projections, facts often project a different reality. Nissan, a major automaker sold 9,819 units of Leaf, an Electric Vehicle (EV) in the USA, against a forecast of 20,000 in 2012. Similarly, improved cookstoves promoted by various governments among the rural and urban underprivileged throughout the world over the past several decades have generally achieved limited success. Other

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such examples abound. Accurate temporal growth forecasts for an innovative product are important for crucial marketing decision variables such as production, product attributes, promotion, distribution, pricing and subsidizing complementary products. This has been shown in the case of innovative products such as satellite radio, e-books and tablet PCs (Moon and Christina, 2002; Ofek, 2005a; Ofek, 2005b).

Research on diffusion of innovations was initiated by Ryan and Gross with the Iowa Corn Studies (Ryan and Gross, 1950). Thereafter, Rogers (1962) developed the Diffusion of Innovation (DoI) theory. Bass (1969) developed a seminal mathematical model that is in conformity with the DoI theory, is parsimonious, and yields good forecasts when adequate and quality diffusion data is available.

The Bass model is a parsimonious mathematical model that is used to forecast the diffusion of a single-purchase innovative product, the principal assumption being that the market is homogeneous. However, some amount of diffusion data is required to calibrate the diffusion curve. Forecasts are generally not very accurate till the peak of the non-cumulative diffusion curve (a bell-shaped curve in time) is reached (Mahajan et al., 1990). The challenge lies in using data available till the takeoff stage or left inflexion point or LIP (Venkatesan and Kumar, 2002), and finding the quantum and timing of the peak and forecast the diffusion thereafter. From theoretical considerations using the Bass model, generally speaking, the takeoff point in the diffusion of an innovative product would be reached earlier in subsistence markets than in developed markets, thus giving us less diffusion data as in-sample data[1]. This is because empirical research suggests that subsistence markets are influenced less by promotionals and more by social network ties (innovation and imitation in the terminology of the Bass model). This is a reason why it can be expected that datasets from subsistence markets can provide a greater challenge and more understanding of the problem of estimation of Bass parameters with data till the LIP. However, a review of the literature on the usage of the Bass model in subsistence markets suggests that data till the left inflexion point is either noisy, or is a short data series or the diffusion curve does not follow the DoI theory fully in that there are kinks in the non-cumulative diffusion curve where diffusion deviates from the DoI theory, and so on. The following paragraph attempts to give an idea of the nature of the problem.

Ratcliff and Doshi (2016) cite Srinivasan and Mason (1986) commenting that, for estimating Bass parameters using their method, at least eight years of

[1]A discussion on this is provided here: https://srdas.github.io/MLBook/productForecastingBassModel.html. If the ratio q/p in the Bass model exceeds 3.59 or so, the time to peak sales and the takeoff time decreases. Since the ratio q/p generally is more in subsistence markets than in developed markets, one expects takeoff time in subsistence markets to be lower.

annual diffusion data are required. Heeler and Hustad (1980) suggest that at least ten years of annual diffusion data are required. This seems to indicate that a certain number of data points are required to accurately estimate Bass parameters. However, for ITC e-Choupal, a product for subsistence markets, only seven years of diffusion data was available till saturation of market potential (Ratcliff and Doshi, 2016). Hence, there was no question of availability of eight to ten years of diffusion data till the left-inflexion point. For Grameenphone in Bangladesh the data shows that the LIP was achieved in year nine, whereas the peak was achieved in year 10, leaving almost no opportunity of finding the peak or estimating the course of diffusion from the LIP. There are also certain instances of sudden drop in non-cumulative diffusion that is not expected under the DoI theory or the Bass model. This yields inaccurate or statistically insignificant results. For instance, Ratcliff and Doshi (2016) report statistically insignificant results for the coefficient of innovation (p) in the cases of both e-Choupal and Grameenphone (Ratcliff and Doshi, 2016). Similarly, Roe-Dale et al. (2015) show that cumulative diffusion of manual-powered irrigation pump-sets falls off at a certain time for data from Kenya. This is inconsistent with the DoI theory. The estimates of Bass parameters exhibit an absurd market potential and the scatter plot of residuals show that estimates are biased in this case. The review also suggests that very little research has been conducted with diffusion data till the takeoff stage with the aim to forecast diffusion thereafter.

Accordingly, this article aims to conduct a review of the literature of estimation techniques of the Bass model and the usage of such techniques in subsistence markets in order to make timely forecasts, preferably at the takeoff stage. The idiosyncrasies of subsistence markets as explained later make them particularly challenging grounds to estimate Bass parameters at the takeoff stage.

The rest of the article is structured as follows: Section 2 provides an idea of diffusion of innovations in general and more specifically in subsistence markets, the setting for this review. Section 3 introduces the Diffusion of Innovations (DoI) theory. Section 4 explains the Bass model, its estimation techniques, and integrates the model with the DoI theory. Section 5 includes a review of the literature of the Bass model in subsistence markets. Section 6 identifies gaps in research and discusses the findings. Section 7 concludes the article.

II. Innovations and Diffusion of Innovations in Subsistence Markets

The question as to what exactly constitutes an innovative product or service is a challenging one. I prefer to use the following definition (White, 1988), which is inclusive:

"development of new products, changes in design of established products, *or use of new materials or components in manufacture of established products.*" (italics applied). As has been rightly commented by White (1988), radical innovations are not necessarily more important than steady, incremental advances in the existing range of products, keeping other marketing mix variables in mind.

In subsistence marketplaces in developing countries such as India, social networks are used by consumers and micro-entrepreneurs as an important enabler for effecting transactions (Viswanathan et al., 2010). Social influence is an important consideration in adoption decisions (such as of mobile phones in subsistence marketplaces in South and South-east Asia including India; De Silva et al., 2009). A few features of innovation strategies in low-income emerging marketplaces stand out (Heeks, 2012):

- Collaborative Innovation, wherein ultimate users are involved in developing the final product
- Grassroots Innovation; a classic example being re-chipping of mobile phones, resulting in low-cost-innards within a high-endbody of a phone
- Frugal Innovation; innovations that are not only low-cost, but also low-demand in other resources. A classic example is the Nokia 1100 mobile phone, described as the "world's best-selling phone".
- Reverse Innovation, wherein ideas flow from emerging markets to the developed markets. An example is the Pingit in the UK, which is modeled on M-Pesa's mobile money transfer model in Kenya.

The above suggests that subsistence markets in emerging economies offer exciting prospects for researchers aiming to study diffusion of new products or services.

An interesting and beneficial innovative product that has failed to achieve the desired degree of success in the rural/subsistence marketplaces in the developing world despite several decades of effort is the improved variety of cookstoves (Slaski and Thurber, 2009). About 2.7 billion people throughout the world depend on biomass for their fuel (Atteridge et al., 2013). This has undesirable consequences for their health, the ambient environment and global climate change. The Chinese National Improved Stove Program (NISP) has been the only success among governmental programs in terms of scale with about 130 million stoves (Shrimali et al., 2011), whereas the Indian National Program on Improved Chulhas (NPIC) has been a near failure with only 32 million stoves sold during the period 1983-2000, with only a fraction of them operational in 1996 (Shrimali et al., 2011). Subsequently, the program has been terminated.

Takeoff is defined as “the first dramatic and sustained increase in a new product’s sales.” The time for takeoff in case of brown goods (entertainment and information products) is an average of two years, whereas the time for takeoff for white goods (kitchen and laundry appliances) is much more, eight years (Tellis, Stremersch, and Yin, 2003 as cited in Chandrasekharan and Tellis, 2007). If we consider improved cookstoves to be a white good, the time for takeoff is long over. Although purchasing power and marketing mix variables across cultures can probably account for this difference to an extent, there certainly remain other reasons as to why cookstoves were poorly adopted.

Among the few pioneering researchers in diffusion of innovations was Everett Rogers, through his seminal book Diffusion of Innovations (Rogers, 1962) that effectively kick-started research on innovation diffusion. Subsequently, Bass (1969) proposed a mathematical model for the diffusion of innovations of one-time buy products in a seminal paper. This opened the floodgates of research on diffusion of innovative products using mathematical models. Rogers (1962; 1983), as the pioneering researcher on diffusion of innovations, has developed a comprehensive theory on the subject. Though the theory has been critiqued, the basic features of the theory have been accepted widely and form the basis of most subsequent research on diffusion of innovations. This theory is introduced in Section 3.

III. The Diffusion of Innovation Theory

A number of seminal theories and models have been proposed by a number of researchers, starting from Rogers (1962), to understand the diffusion of innovations and factors that influence user acceptance of such innovations. One of the earliest and most comprehensive theories on the diffusion of innovations was propounded by Rogers (1962). Although Gabriel Tarde, a French sociologist, was the first to indicate the nature of the cumulative diffusion curve (S-shaped) in 1903 (Rogers, 1976) and the first modern study on diffusion (of hybrid corn among Iowa farmers, described as a “revolutionary paradigm”), was published by two sociologists, Ryan and Gross, in 1943, Rogers is considered to be the seminal theorist in the area of diffusion of innovations.

The theory says that the non-cumulative diffusion curve in time resembles a normal distribution (bell-shaped) curve, while the cumulative diffusion curve is S-shaped (Figure 1 and Figure 2 respectively). Adopters are divided into five groups based on the time of adoption: innovators (2.5%), early adopters (13.5%), early majority (34%), late majority (34%) and laggards (16%). Subsequent mathematical models have shown the validity of such assertions.

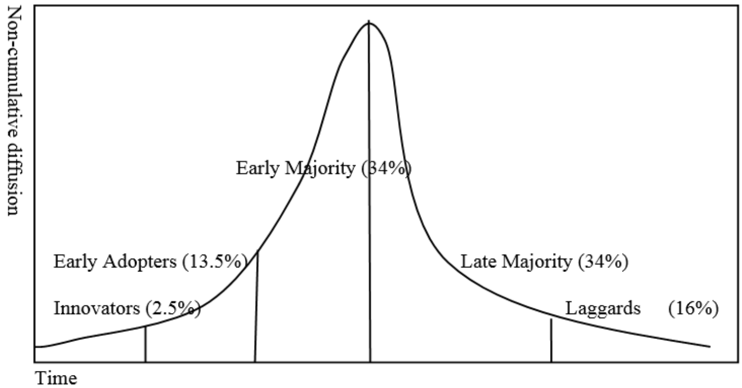

Figure 1 Non-cumulative diffusion curve of Rogers

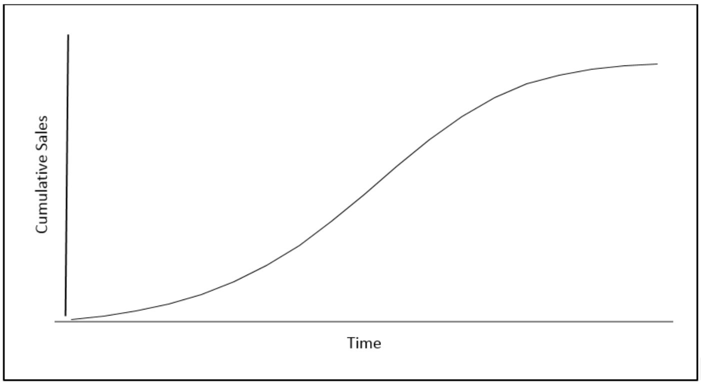

Figure 2 Cumulative diffusion curve

What is diffusion? Webster (2004) defines the noun "diffusion" as the "spread of a cultural or technological practice or innovation from one region to another, as by trade or conquest, widely." Economics defines diffusion as "the spread of an innovation across social groups over time." (Stoneman, 2002 as cited in Chandrasekharan and Tellis, 2007). However, in contrast to the above, in marketing and communication, the key element is "population/members of a social system." Rogers (1983, p.5) defined diffusion of innovations as the process by which an innovation is "communicated through certain channels over time among members of a social system," thus implying that the four key elements in the process are: innovation, communication channels, time, and social system. A more contemporary definition is: "Innovation diffusion is the process of the market penetration of new products and services driven by social influences. Such influences include all the interdependencies among consumers that affect various market players, with and without their explicit knowledge." (Peres et al., 2010). Originating broadly in the domain of rural sociology, the Diffusion of Innovation (hereinafter DoI) theory has since been embraced by the marketing literature since the 1960s (Mahajan et al., 1990).

IV. The Diffusion of Innovation Theory and the Bass Model of Diffusion

The Bass model (1969), developed by Frank Bass (1926-2006), a pioneer in marketing science, is a parsimonious and quite accurate model to forecast the diffusion of innovations, and the model and its variants are in wide use. Given three parameters: p (the coefficient of innovation), q (the coefficient of imitation) and m (the market potential), this mathematical model can estimate sales trajectory of a single-purchase new product.

The theory on innovation diffusion, due to Rogers (1962; 1983), as mentioned earlier, considers the following classes of adopters of a new product: innovators, early adopters, early majority, late majority, and laggards. Innovators are defined as individuals who decide on the adoption of a new product independent of social pressures while others are influenced in the timing of adoption by social pressures (Bass, 1969). The literature on aggregate mathematical models on innovation diffusion is extensive and most of it uses the Bass model or a variant.

The Bass model broadly follows the terminology of Rogers (1983). Adopters are classified into one of two groups (Mahajan et al., 1990). The first group is influenced only by mass media communications ("innovators" in Bass terminology) while the second group is influenced only by word-of-mouth ("imitators" in Bass terminology). In its most basic form, the Bass model is the outcome of a hazard function, i.e., the probability that an adoption will take place at time t given the fact that it has not yet taken place. Mathematically,

$$f(t)/(1-F(t)) = p + qF(t) \qquad (1)$$

where f(t) is the density function in time to adoption, F(t) is the cumulative fraction of adopters in time t, and p and q are known as the coefficient of innovation and the coefficient of imitation respectively (Bass, 1969; Mahajan et al, 1990). The expression on the left hand side is known as hazard function. If the potential number of adopters (or in other words, the market size) is m, then the cumulative number of adopters is mF(t) = N(t), where N(t) is the cumulative sales at time t and n(t) is the sales at time t. Mathematically, after a little algebraic manipulation (Mahajan et al, 1990),

$$n(t) = dN(t)/dt = p[m - N(t)] + (q/m)*N(t)[m-N(t)] \qquad (2)$$

This clearly shows that any adoption is the outcome of either of the two processes: “innovativeness”, which is represented by the factor p, and “learning”, which is represented by the factor q.

The cumulative sales N(t) at a given point of time t is given by Mahajan et al., 1990:

$$N(t) = m[(1 - e^{-(p+q)t})/(1 + (q/p)e^{-(p+q)t})] \qquad (3)$$

1. Parameter Estimation

Estimation of the parameters p, q, and m is necessary to forecast sales. Since the Bass model involves estimation of three parameters p, q, and m, adoption data for a minimum of three periods are necessary for estimation. Empirical research, however, has indicated that robust and stable estimates of parameters are possible only when the data under consideration is inclusive of the peak of the non-cumulative diffusion curve (Heeler and Hustad, 1980; Srinivasan and Mason, 1986).

A number of estimation procedures have been proposed by researchers for estimation of parameters. Bass (1969) used the ordinary least squares (OLS) procedure. He took the regression (or discrete) analog of the differential equation formulation of the Bass model (equation 2) that yielded:

$$N(t+1) - N(t) = pm + (q-p)N(t) - (q/m)\, N^2(t)$$
$$\text{or, } n(t+1) = a + bN(t) + cN^2(t) \qquad (4)$$

where $a = pm$, $b = (q - p)$ and $c = -(q/m)$. OLS can then be used to estimate a, b and c in equation (3). Thus estimation of p, q, and m is possible.

However, the OLS suffers from three limitations (Schmittlein and Mahajan, 1982), namely, multicollinearity between N(t) and $N^2(t)$; no standard errors are available for estimated p, q, and m thus giving no idea about their statistical significance; and, time-interval bias that emanates from the fact that discrete time-series data are used for estimating a continuous model.

To alleviate these shortcomings, Schmittlein and Mahajan (1982) proposed a Maximum Likelihood Estimation (MLE) process to estimate the parameters from the solution of the differential equation. This method has limitations as well. Srinivasan and Mason (1986) point out that the MLE suffers from the following limitations: it considers sampling errors and ignores all other errors, such as the effects of marketing variables on the diffusion process, and it underestimates the standard errors of the estimated parameters that results in erroneous inferences of the statistical significance of these parameters.

Srinivasan and Mason (1986), hence, suggested the Nonlinear Least Squares (NLS) method to estimate the parameters. In addition to overcoming the time-interval bias in the OLS procedure, here the error term represents the effect of sampling errors, excluded marketing variables and the misspecification of the density function. Hence the standard errors for the parameters are expected to be more accurate.

More recently, certain advanced techniques have been proposed that are designed to provide more accurate forecasts. Typically, accurate forecasts using the Bass model are possible when data beyond the two infection points, takeoff and slowdown, are available. However, when this is not available, diffusion data for similar or dissimilar products can be used. Two crucial questions are: can products be classified as similar or dissimilar? And what can be done when products are dissimilar? To answer the second question, some researchers have proposed to use Hierarchical Bayesian methods to model the diffusion of new products. Information from some products that share certain common structures is used to develop prelaunch forecasts for the focal product, with the subsequent updating of such forecasts as sales data from the focal product becomes available, leading to more stable forecasts (Lenk and Rao, 1990; Talukdar et al., 2002).

To draw more realistic estimates, some researchers have used (adaptive) stochastic techniques that permit parameters to vary with time to model new product diffusion. Feedback filters and Bayesian techniques are used in these methodologies to update parameters over time (Bretschneider and Mahajan, 1980; Xie et al., 1997).

The use of genetic algorithms, a technique that combines the advantages of both sequential search-based and random search to estimate parameters of the Bass model has been proposed (Venkatesan and Kumar, 2002; Venkatesan et al, 2004; Wenrong et al., 2006). The method enjoys a greater chance of reaching the global optimum as compared to sequential search-based methods. The method, according to the authors, provides more accurate and realistic estimates of parameters as compared to sequential search based methods.

Table 1 Techniques to estimate the Bass model with some diffusion data

Technique	Author(s) and Year	Merits and Disadvantages
Ordinary Least Squares (OLS)	Bass (1969)	Simple to estimate; time-invariant parameters Discrete analog required Multicollinearity between N(t) and $N^2(t)$ No standard errors are available for estimated p, q, and m thus giving no idea about their statistical significance Time-interval bias that emanates from the fact that discrete time-series data are used for estimating a continuous model
Adaptive Filter	Bretschneider and Mahajan (1980)	Discrete analog of Bass equation required Time-varying parameters
Maximum Likelihood Estimator (MLE)	Schmittlein and Mahajan (1982)	Considers sampling errors and ignores all other errors, such as the effects of marketing variables on the diffusion process; underestimates the standard errors of the estimated parameters that results in erroneous inferences of the statistical significance of these parameters Time-invariant parameters
Non-linear Least Squares (NLS)	Srinivasan and Mason (1986)	Overcomes the time-interval bias in the OLS procedure The error term represents the effect of sampling errors, excluded marketing variables and the misspecification of the density function. Hence the standard errors for the parameters are expected to be more accurate. Time-invariant parameters
Bayesian Updates in Meta-analysis	Sultan, Farley and Lehmann (1990)	Produces more robust estimates than OLS and WLS, particularly early on in diffusion Analytical solution of Bass equation required
Hierarchical Bayesian Analysis (HBA)	Lenk and Rao (1990)	Time-varying parameters Analytical solution required Forecasts on a product dependent on the diffusion of different products that share commonalities. Forecasts at time zero (start) is similar to pooled forecasts with forecasts adapting to reported sales in time.
Adaptive Kalman Filter with Continuous State and Discrete Observations AKF (C-D)	Xie et al. (1997)	Time-varying parameters; can be used for estimating parameters that change with time Analytical solution or discrete analog not required Explicitly incorporates observation errors in the estimation process Algorithm is relatively easy to implement
Combination of Non-linear Least Squares (NLS), a stationary stochastic procedure (using the Kalman filter), and a non-stationary stochastic model specification (the Cooley-Prescott procedure)	Putsis (1998)	Time-varying parameters Analytical solution or discrete analog not required for the stochastic estimation procedures Stochastic estimation techniques are much more efficacious than non-stochastic procedures Techniques assuming non-stationary behavior show marginally better results than techniques assuming stationary behavior
Genetic Algorithms (GA)	Venkatesan, Krishnan and Kumar (2004); Venkatesan and Kumar (2002)	Time-invariant parameters Analytical solution required Expected to reach global optima Computation time and algorithmic complexity an issue Typically reaches a point close to the global optima, and can be used as a complementary search technique to gradient-based methods such as NLS Generally empirical evidence shows that it performs better than the AKF (C-D) and Sequential Search Based-Nonlinear methods No guarantee of reaching the global optima. Results depend partially on the initial estimates provided.
Simulated Annealing (SA)	Mitra (2018)	Time-invariant parameters Analytical solution required Expected to reach global optima Typically reaches a point close to the global optima, and can be used as a complementary search technique to gradient-based methods such as NLS. No guarantee of reaching the global optima.

Simulated Annealing (SA) is a general-purpose serial algorithm used to locate the global optima as opposed to local optima of a continuous function (Du and Swamy, 2016). The method, which has been inspired by the statistical mechanics of annealing, (heating and subsequent slow cooling to obtain good crystals) often uses the Metropolis algorithm for simulation (Rutenbar, 1989). This method has been used by Mitra (2018) with encouraging results to estimate Bass parameters in the case of the diffusion of mobile telephony in subsistence markets in India.

It has been suggested that the challenge to diffusion forecasters is to estimate Bass model parameters with limited diffusion data till the Left Inflexion Point (LIP). The LIP is the point where the non-cumulative diffusion curve exhibits maximum slope. It is given by

$$T_{left}^{**} = \frac{1}{(p+q)} \ln\left(\frac{q}{p}\right)\left(2-\sqrt{3}\right)$$

This represents the takeoff period for an innovative product and data till this point is required for forecasting diffusion including the diffusion peak. This point is taken as a benchmark in the literature (Venkatesan and Kumar, 2002). Table 1 exhibits various techniques for the Bass model estimation.

2. Merits and Demerits of the Bass Model

The Bass model has several attributes that appeal to researchers (Chandrasekharan and Tellis, 2007):

- In this model, sales is a quadratic function of prior cumulative sales, and hence fits the S-curve typical of sales of new products well. Subsequent more complicated refinements could not show much improvement on the basic model, and as such the model is parsimonious.
- The interpretations of the parameters p (the coefficient of innovation that reflects the spontaneous rate of adoption) and q (the coefficient of imitation that reflects the effect of prior cumulative adopters on innovation) have strong behavioral connotations.
- The time to, and magnitude of peak sales are important parameters to marketing managers. The Bass model provides clear answers to these parameters.
- The two special cases of p=0 and q=0 are interesting. When q=0, the Bass model reduces to an exponential function driven by innovation only (Fourt and Woodlock, 1960). On the other hand, when p=0, the model reduces to a logistic diffusion function, running on imitative

processes (Fisher and Pry, 1972). So, the Bass model is a generalized model.

The Bass model has gained popularity for good reason. However, the basic Bass model has drawbacks (Lamberson, 2008; Massiani, 2013). These are:

- Estimates of parameters p, q, and m are generally made using past data. This, however, does not permit forecasting using limited data or before the actual launch of the product. In such cases managerial judgment or historical analogues are used.
- Robust and stable estimates of the parameters p, q, and m are possible only when the data available is inclusive of the peak of the non-cumulative adoption curve (Heeler and Hustad 1980; Srinivasan and Mason 1986 as cited in Mahajan et al., 1990). This is a strong drawback of the model that this article attempts to address.
- Certain assumptions in the model are questionable:
    - Constancy of market potential
    - Diffusion of an innovation is not affected by diffusion of other innovations
    - Absence of supply restrictions
    - Absence of repeat or replacement purchases
    - The view of diffusion as a binary process (adoption/non-adoption), thus disregarding the stages in the adoption process
    - For certain industries, such as the auto-industry, the issue of cumulative sales is a vexed one. The total time period to be considered is difficult to judge

3. Error Margins in the Literature

Error margins in typical forecasting exercises with the Bass model are high. Mean Absolute Percentage Error (MAPE) varies between 5% and 10% in time horizons of less than or equal to three years (Meade and Islam, 2015). Generally, MAPE values lesser than 20% are acceptable (Hwang et al., 2009 as cited in Avila et al., 2017). The following Table 2 adapted from Xie, Song, Sirbu and Wang (1997) lists 1-step-ahead errors for seven new products. Table 2 provides an idea of the error margins in the forecasting literature using the Bass model on pre-peak data that are capable of making timely forecasts. OLS and NLS were unable to make any statistically significant forecasts with pre-peak data in the research cited.

Table 2 Typical error margins using the Bass model on Pre-peak data, Adapted from Xie, Song, Sirbu and Wang (1997)

Mean Absolute Percentage Error (MAPE) using Estimation Method	Products						
	AC[3]	CTV[4]	Clothes Dryers	USG[5]	Mammo -graphy	FL[6]	AP[7]
AF[1]	51.3	71.3	47.5	50.1	40.5	48.5	83.1
AKF (C-D)[2]	40.1	33.0	35.2	56.5	27.8	50.1	85.5

[1]Adaptive Filter, [2]Adaptive Kalman Filter with Continuous State and Discrete Observations, [3]Air Conditioners, [4]Color TVs, [5]Ultrasonography,[6]Foreign Language,[7]Accelerated Program.

V. The Bass Model and Related Models in Subsistence Markets

One sector where early diffusion studies were conducted is agriculture including the seminal study by Ryan and Gross (1950) on hybrid corn diffusion. This was followed by another seminal work by Griliches (1957), which used mathematical modeling to study differences in the rates of usage of hybrid corn seeds in the USA. Subsequently, many authors have used the Bass model or its variants to study diffusion of technology in agriculture, in India and elsewhere. I have identified eight studies that have used the Bass model to study diffusion in subsistence markets that are relevant to the research objectives of the current paper.

Understanding the Diffusion of Innovation in Subsistence Markets Using the Bass or Similar Models

One area that has been highlighted as a fertile ground for further research is the diffusion of innovations in developing countries and emerging economies (Muller, Peres and Mahajan, 2009, p.77). Although there has been a reasonable amount of research in cross-country influences and cross-country growth patterns, only limited research using mathematical models had focused on unique patterns of diffusion of innovations in subsistence markets, with the exception of a few papers. The following Table 3 presents the details of such research in a structured manner.

Table 3 The Bass or similar models in rural or subsistence markets

Author(s)	Year	Journal	Research Questions/Hypotheses/Purpose of Study	Model	Data and Analysis Methodology	Results	Remarks Relevant to the Current Research
Griliches	1957	Econometrica	Understanding factors influencing the wide differences in usage rates of hybrid seed corn In the USA	Logistic growth model	Data of corn acreage planted with hybrid seed for various US states available from USDA was fitted with a logistic curve. The three parameters of the logistic curve: origin, slope and ceiling were explained in terms of economic variables.	Widely cited study, one of the first to use mathematical modeling to understand diffusion of an innovation. Lag in entry of seed producers in particular areas were explained in terms of profitability of entry. Differences in long-run equilibrium and rates of approach to that equilibrium were explained in terms of difference of profitability emanating from a shift from open-pollinated to hybrid seeds.	Annual data for about 24 years fitted to a logistic curve. No forecasts.
Akinola	1986	Journal of Agricultural Economics	To ascertain whether the Bass model performs better than a purely imitative model Make forecasts	Bass model and an imitative model	Annual time series data (1955-80) of adopters of cocoa-spraying chemicals in Nigeria was used. The Bass model was fitted with the data. The non-linear model was linearized.	The Bass model performed marginally better than the imitative model $p = 0.00451$, $q = 0.03551$	Large in-sample dataset (1955-76) inclusive of peak. Out-of-sample dataset: 1977-80. No error diagnostics.
Gore and Lavaraj	1987	Technological Forecasting and Social Change	Proposing and testing a model when complete intermixture of prior and potential adopters is not a valid assumption	Logistic model and an internal communication model	Population from a town near Poona, India and surrounding villages was considered. The diffusion of crossbred goats was studied in the two populations (the town and the villages)	Fit statistics improved when using a combination of the logistic and the new model as against the logistic model	In-sample dataset 1976-84; the entire data available. No holdout data.
Purohit and Kandpal	2005	Renewable and Sustainable Energy Reviews	Future diffusion levels of four renewable energy technologies for irrigation water pumps in India Estimates of investment required for this technology diffusion	Bass, Gompertz, Logistic and Pearl models	Data from Ministry of Non-Conventional Energy Sources, Government of India, was used to estimate parameters of the Bass model. Parameters for other models were estimated. Diffusion was forecast for the four technologies till 2025.	q is highest for biogas-driven pump. Diffusion of renewable energy technologies for irrigation pumps is not likely to reach its potential by 2025. Diffusion figures from the Bass model are lower than that from the Logistic and Pearl models.	In-sample and out-of sample data not very clearly mentioned. Forecast errors not mentioned.
McRoberts and Franke	2008	Working Paper Number 29. Land Economy Working Paper Series.	The purpose of the study was to build a diffusion model that takes into	A model that takes into account the effect of aggregation (ecological	Data from 25 villages in Haryana state, India, in districts that practiced zero-tillage method of wheat	An increment in the level of aggregation among adopters leads to an increment in the time taken to achieve the market	Model estimation. Some forecasts but no model evaluation.

		Land Economy Research Group, Edinburgh	consideration the aggregation in adopters and non-adopters either physically and/or culturally.	term) in the adopting population on spatial or cultural basis	cultivation was used to test the model.	potential. This also leads to an increase in the maximum rate of adoption.	
Ratcliff and Doshi	2016	Business and Society	On theoretical grounds in BoP markets: H1: p is lower than that in developed markets H2: q is higher than that in developed markets H3: Innovations with low barriers to entry to entry or high trialabilty would have higher values of both p and q	Bass model	Data from three innovative programs: Grameen's Village Phone (Bangladesh), Patrimonio Hoy (Mexico), and ITC e-Choupal (India) were fitted to the Bass model using the NLS method	H1 is substantially established H2 is substantially established In case of H3, evidence is mixed Patrimonio Hoy seems to be an exception. Empirical evidence from the Bass model is useful in resolving cases where theoretical considerations provide no clear answer. Patrimonio Hoy: p=.030; q=0.36; e-Choupal: $p=7.6*10^{-8}$; q=3.88; Village Phone: $1.1*10^{-4}$; q=0.85	The entire time-series is taken as in-sample data. In case of Village Phone the peak was reached; going by the methodology the diffusion was over for ITC e-Choupal and the first peak was achieved for Patrimonio Hoy. Even then Bass parameters were not statistically significant in some cases. Specifically, statistically insignificant results for the coefficient of innovation (p) in the cases of both e-Choupal and Grameenphone were reported. Forecasts have not been attempted. No error diagnostics.
Guo and Liu	2014	Computer Modelling and New Technologies	Estimating, ex-post, the pattern of sales of home appliances in rural China during December-January 2009	Bass model	Secondary data from home appliances sales in rural China during Dec-Jan 2009 was fitted to the Bass model using linear least squares, non-linear least squares, and Bayesian parameter estimation methods	The Bayesian parameter estimate yields the closest results to the historical data. p=0.01293; q=0.3044 using Bayesian Parameter Estimation.	The entire monthly time series of January 2009-December 2009 has been taken as the in-sample data. No forecasts were attempted.
Roe-Dale, Brown and Staton	2015	Int. J. of Social Entrepreneurship and Innovation	Is the Bass model appropriate for BoP markets? Estimation of market potential Determination of effective marketing strategies in BoP markets	Bass model	Data relating to Manual Irrigation Pimps (MIPs) were analyzed from Bangladesh, Kenya and Tanzania	The Bass model was found to be appropriate for Bangladesh and Tanzania, but not for Kenya. For B'desh: p = .008546; q = .28185 For Kenya: p = .0015; q = .0911 For Tanzania: p = .0061, and q = .308	The estimates of Bass parameters exhibit an absurd market potential and the scatter-plot of residuals show that estimates are biased in the case of Kenyan data. In-sample data not clearly mentioned. No error diagnostics were attempted.

VI. Discussion

Table 3 reveals certain patterns. Only one article out of the eight used part of the dataset as holdout. All other papers used the entire dataset to estimate the model. In many cases, the dataset is large, so that the LIP has been exceeded by a long margin. The non-linear method is the predominantly used method, although one article used Bayesian estimation methods. None of the articles (excepting one partially) had the appropriate choice of model estimation technique as a research question or purpose of study. Error diagnostics on forecast data have not been attempted in any article.

Thus, the above indicates that there really is no effective research on the estimation of diffusion model (Bass/Gompertz/Logistic) parameters at an early stage, closer to the LIP. Given the issues inherent in diffusion modeling in rural/subsistence marketplaces, there exists scope for introducing improved estimation techniques in rural/subsistence marketplaces. The literature reveals that, even when the entire dataset was used to estimate parameters, in some cases estimates were found to be statistically insignificant: in case of Village Phone, Bangladesh, and ITC e-Choupal, India (Ratcliff and Doshi, 2016). This seems to indicate that the non-linear regression that is the mainstay of such estimation processes does not work effectively with noisy data series that are often the characteristics of diffusion in subsistence and emerging marketplaces.

Boateng (2016) defines research gaps as “discrepancies in existing research literature which need to be addressed.” He classifies research gaps as one of the following:

- Issue Gap: This exists when an issue is under-represented in the literature. The issue of estimation of Bass parameters with diffusion data till the takeoff stage has generally been neglected with a few articles being the exceptions. Most articles do not design research in a way that requires them to evaluate the model with a large out-of-sample dataset. This issue has been acknowledged in the literature (Mahajan, Muller and Bass, 1990), but only very limited research attempts to bridge the gap.
- Method Gap: This exists when a research methodology is under-represented in the literature with reference to a research issue. Inadequate or conflicting empirical results may point to method gaps. Sequential search-basedtechniques such as NLS has generally been the de-facto standard in estimation of Bass parameters. Bayesian techniques, filter theory, random search techniques have been exceptions to this rule.
- Context Gap: This exists when research contexts such as sector, industry or spatial regions are under-represented in the literature with

reference to a particular research issue. Muller, Peres and Mahajan (2009, p.77-78) acknowledge that there exist research gaps with regard to diffusion in the developing world. The diffusion of innovative products in subsistence markets with their own challenges in terms of idiosyncrasies in data and consequent model estimation methodologies have been an under-researched area.

The "Fourth Industrial Revolution," wherein boundaries between physical, digital and biological spaces are blurring, has been the subject of academic discussion recently. Jeon and Suh (2017) find that frequently-used keywords in the relevant literature are AI, Internet, smart, data, system and digital. These are all intimately associated with technology innovations. Muller et al. (2009) clearly state that the Bass model is very appropriate for the diffusion of technology products. To that extent, this review is important for creating a research agenda to understand the Fourth Industrial Revolution in subsistence and emerging markets, a process that is already underway to a small extent.

VII. Conclusion

The estimation of the Bass model with data till the takeoff stage is a challenging exercise. This has generally been attempted only in a few cases and, even with sophisticated estimation methods, is a difficult task. Subsistence markets, with their own set of idiosyncrasies in data, present challenges to the marketing scientist in this regard. In this context, this article attempted to take a look at the extant literature on the use of the Bass model in subsistence markets to provide timely forecasts in the context of diffusion of innovative products. In this exercise, the article introduced the diffusion of innovation theory, diffusion in subsistence markets, the Bass model and its estimation techniques with their relative strengths and weaknesses. Thereafter, a review of the use of the Bass model in subsistence markets was conducted that delineated research gaps. The research discussed above showed that timely forecasts at the takeoff stage or at least prior to the peak have almost never been attempted in subsistence marketplaces. Even with complete diffusion datasets, the existing model estimation techniques have yielded statistically insignificant parameter estimates in some cases. Thus, there is a need to revamp model estimation techniques and use them to provide timely forecasts at the takeoff stage or at any rate prior to the attainment of peak. The review of literature on the estimation techniques juxtaposed with the literature on the use of the Bass model in subsistence markets indicates that there is scope for making timely forecasts at the takeoff stage by using estimation techniques that uses random search algorithms, filter theory or Bayesian algorithms. Future research might focus on timely estimation of Bass parameters using these methods in subsistence markets that are expected to see an array of innovative product launches for whom timely and accurate forecasts would be necessary.

Note from Dr Shashi Jain, Chair of COSMAR 2018

A key challenge for a marketing team while introducing an innovative product in a new market is to forecast the diffusion or the adoption of the product by the market. While traditionally Bass model has widely been used to forecast diffusion of innovative products, using it to forecast adoption of innovative products in subsistence market can be challenging. The current paper does an in-depth review of the methodologies employed to forecast diffusion of innovative products in general, and with respect to subsistence markets in particular.

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